

Zeta Nonlocal Scalar Fields

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Abstract

We consider some nonlocal and nonpolynomial scalar field models originated from p -adic string theory. Infinite number of spacetime derivatives is determined by the operator valued Riemann zeta function through d'Alembertian \square in its argument. Construction of the corresponding Lagrangians L starts with the exact Lagrangian \mathcal{L}_p for effective field of p -adic tachyon string, which is generalized replacing p by arbitrary natural number n and then taken a sum of \mathcal{L}_n over all n . The corresponding new objects we call zeta scalar strings. Some basic classical field properties of these fields are obtained and presented in this paper. In particular, some solutions of the equations of motion and their tachyon spectra are studied. Field theory with Riemann zeta function dynamics is interesting in its own right as well.

*Dedicated to Vasiliy Sergeevich Vladimirov
on the occasion of his 85th birthday*

1 Introduction

Already two decades have passed since the first paper on a p -adic string was published [1]. So far various p -adic structures have been observed not only in

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string theory but also in many other models of modern mathematical physics (for a review of the early days developments, see e.g. [2, 3]).

One of the greatest achievements in p -adic string theory is an effective field description of open scalar p -adic strings [4, 5]. The effective tachyon Lagrangian is very simple and exact. It describes not only four-point scattering amplitudes but also all higher ones at the tree-level.

This p -adic string theory has been significantly pushed forward when was shown [6] that it describes tachyon condensation and brane descent relations simpler than by ordinary bosonic strings. After this success, many aspects of p -adic string dynamics have been investigated and compared with dynamics of ordinary strings (see, e.g. [7, 8, 9, 10] and references therein). Noncommutative deformation of p -adic string world-sheet with a constant B-field was investigated in [11] (on p -adic noncommutativity see also [12]). A systematic mathematical study of spatially homogeneous solutions of the relevant nonlinear differential equations of motion has been of considerable interest (see [9, 13, 14, 15] and references therein). Some possible cosmological implications of p -adic string theory have been also investigated [16, 17, 18, 19, 20]. It was proposed [21] that p -adic string theories provide lattice discretization to the world-sheet of ordinary strings. As a result of these developments, some nontrivial features of ordinary string theory have been reproduced from the p -adic effective action. Moreover, there have been established so far many similarities and analogies between p -adic and ordinary strings.

Adelic approach to the string scattering amplitudes is a very useful way to connect p -adic and ordinary counterparts (see [2, 3] as a review). Moreover, it eliminates unwanted prime number parameter p contained in p -adic amplitudes and also cures the problem of p -adic causality violation. Adelic generalization of quantum mechanics was also successfully formulated, and it was found a connection between adelic vacuum state of the harmonic oscillator and the Riemann zeta function [22]. Recently, an interesting approach toward foundation of a field theory and cosmology based on the Riemann zeta function was proposed in [23]. Note that p -adic and ordinary sectors of the four point adelic string amplitudes separately contain the Riemann zeta function (see, e.g. [2], [3] and [24]).

Main motivation for the present paper is our intention to obtain the corresponding effective Lagrangian for adelic scalar string. Hence, as a first step we investigate possibilities to derive Lagrangian related to the p -adic sector of adelic string. Starting with the exact Lagrangian for the effective field of p -adic tachyon string, extending prime number p to arbitrary natural

number n and undertaking various summations of such Lagrangians over all n , we obtain some scalar field theories with the operator valued Riemann zeta function. Emergence of the Riemann zeta function at the classical level can be regarded as its a counterpart in quantum scattering amplitude. As we shall see this zeta function controls spacetime nonlocality. In the next sections we construct and explore some classical field models which may be regarded as candidates for description of some properties of an adelic open scalar string.

2 Modeling of some zeta nonlocal scalar fields

The exact tree-level Lagrangian of effective scalar field φ , which describes open p -adic string tachyon, is

$$\mathcal{L}_p = \frac{m_p^D}{g_p^2} \frac{p^2}{p-1} \left[-\frac{1}{2} \varphi p^{-\frac{\square}{2m_p^2}} \varphi + \frac{1}{p+1} \varphi^{p+1} \right], \quad (1)$$

where p is any prime number, $\square = -\partial_t^2 + \nabla^2$ is the D -dimensional d'Alembertian and we adopt metric with signature $(- + \dots +)$. An infinite number of space-time derivatives follows from the expansion

$$p^{-\frac{\square}{2m_p^2}} = \exp \left(-\frac{1}{2m_p^2} \ln p \square \right) = \sum_{k=0}^{+\infty} \left(-\frac{\ln p}{2m_p^2} \right)^k \frac{1}{k!} \square^k.$$

The equation of motion for (1) is

$$p^{-\frac{\square}{2m_p^2}} \varphi = \varphi^p, \quad (2)$$

and its properties have been studied by many authors (see e.g. [9, 13, 14, 15] and references therein).

It is worth noting that prime number p in (1) and (2) can be replaced by any natural number $n \geq 2$ and such expressions also make sense. Moreover, when $p = 1 + \varepsilon \rightarrow 1$ there is the limit of (1)

$$\mathcal{L} = \frac{m^D}{g^2} \left[\frac{1}{2} \varphi \frac{\square}{m^2} \varphi + \frac{\varphi^2}{2} (\ln \varphi^2 - 1) \right] \quad (3)$$

which is related to the ordinary bosonic string in the boundary string field theory [25].

Now we want to introduce a model which incorporates all the above string Lagrangians (1) with p replaced by $n \in \mathbb{N}$. To this end, we take the sum of all Lagrangians \mathcal{L}_n in the form

$$L = \sum_{n=1}^{+\infty} C_n \mathcal{L}_n = \sum_{n=1}^{+\infty} C_n \frac{m_n^D}{g_n^2} \frac{n^2}{n-1} \left[-\frac{1}{2} \phi n^{-\frac{\square}{2m_n^2}} \phi + \frac{1}{n+1} \phi^{n+1} \right], \quad (4)$$

whose explicit realization depends on particular choice of coefficients C_n , masses m_n and coupling constants g_n . To avoid a divergence problem of $1/(n-1)$ when $n=1$ one has to take that $C_n m_n^D/g_n^2$ is proportional to $n-1$. In this paper we shall consider a case when coefficients C_n are proportional to $n-1$, while masses m_n as well as coupling constants g_n do not depend on n , i.e. $m_n = m$, $g_n = g$. Since this is an approach towards effective Lagrangian of an adelic string there is a sense to take mass and coupling constant independent on particular p or n . Namely, it seems to be natural that an adelic physical object has fixed rational valued parameters. To emphasize that Lagrangian (4) describes a new field, which is different from a particular p -adic one, we introduced notation ϕ instead of φ . The two terms in (4) with $n=1$ are equal up to the sign, but we remain them because they provide the suitable form of total Lagrangian L .

We shall consider a simple case

$$C_n = \frac{n-1}{n^{2+h}}, \quad (5)$$

where h is a real number. The corresponding Lagrangian reads

$$L_h = \frac{m^D}{g^2} \left[-\frac{1}{2} \phi \sum_{n=1}^{+\infty} n^{-\frac{\square}{2m^2}-h} \phi + \sum_{n=1}^{+\infty} \frac{n^{-h}}{n+1} \phi^{n+1} \right] \quad (6)$$

and it depends on parameter h .

According to the famous Euler product formula one can write

$$\sum_{n=1}^{+\infty} n^{-\frac{\square}{2m^2}-h} = \prod_p \frac{1}{1 - p^{-\frac{\square}{2m^2}-h}}.$$

Recall that standard definition of the Riemann zeta function is

$$\zeta(s) = \sum_{n=1}^{+\infty} \frac{1}{n^s} = \prod_p \frac{1}{1-p^{-s}}, \quad s = \sigma + i\tau, \quad \sigma > 1, \quad (7)$$

which has analytic continuation to the entire complex s plane, excluding the point $s = 1$, where it has a simple pole with residue 1. Employing definition (7) we can rewrite (6) in the form

$$L_h = \frac{m^D}{g^2} \left[-\frac{1}{2} \phi \zeta\left(\frac{\square}{2m^2} + h\right) \phi + \sum_{n=1}^{+\infty} \frac{n^{-h}}{n+1} \phi^{n+1} \right]. \quad (8)$$

Here $\zeta\left(\frac{\square}{2m^2} + h\right)$ acts as a pseudodifferential operator

$$\zeta\left(\frac{\square}{2m^2} + h\right) \phi(x) = \frac{1}{(2\pi)^D} \int e^{ixk} \zeta\left(-\frac{k^2}{2m^2} + h\right) \tilde{\phi}(k) dk, \quad (9)$$

where $\tilde{\phi}(k) = \int e^{-ikx} \phi(x) dx$ is the Fourier transform of $\phi(x)$. Lagrangian L_0 , with the restriction on momenta $-k^2 = k_0^2 - \vec{k}^2 > (2-2h)m^2$ and field $|\phi| < 1$, is analyzed in [26]. In the sequel we shall consider Lagrangian (8) with analytic continuations of the zeta function and the power series $\sum \frac{n^{-h}}{n+1} \phi^{n+1}$, i.e.

$$L_h = \frac{m^D}{g^2} \left[-\frac{1}{2} \phi \zeta\left(\frac{\square}{2m^2} + h\right) \phi + \mathcal{AC} \sum_{n=1}^{+\infty} \frac{n^{-h}}{n+1} \phi^{n+1} \right], \quad (10)$$

where \mathcal{AC} denotes analytic continuation.

Nonlocal dynamics of this field ϕ is encoded in the pseudodifferential form of the Riemann zeta function. When the d'Alembertian is in the argument of the Riemann zeta function we shall say that we have zeta nonlocality. Consequently, the above ϕ is a zeta nonlocal scalar field.

Potential of the above zeta scalar field (10) is equal to $-L_h$ at $\square = 0$, i.e.

$$V_h(\phi) = \frac{m^D}{g^2} \left(\frac{\phi^2}{2} \zeta(h) - \mathcal{AC} \sum_{n=1}^{+\infty} \frac{n^{-h}}{n+1} \phi^{n+1} \right), \quad (11)$$

where $h \neq 1$ since $\zeta(1) = \infty$. The term with ζ -function vanishes at $h = -2, -4, -6, \dots$.

The equation of motion in differential and integral form is

$$\zeta\left(\frac{\square}{2m^2} + h\right) \phi = \mathcal{AC} \sum_{n=1}^{+\infty} n^{-h} \phi^n, \quad (12)$$

$$\frac{1}{(2\pi)^D} \int_{\mathbb{R}^D} e^{ikx} \zeta\left(-\frac{k^2}{2m^2} + h\right) \tilde{\phi}(k) dk = \mathcal{AC} \sum_{n=1}^{+\infty} n^{-h} \phi^n, \quad (13)$$

respectively. Obviously $\phi = 0$ is a trivial solution for any real h . Existence of other trivial solutions depends on parameter h . When $h > 1$ we have another trivial solution $\phi = 1$.

In the weak field approximation ($|\phi(x)| \ll 1$) the above expression (13) becomes

$$\int_{\mathbb{R}^D} e^{ikx} \left[\zeta\left(-\frac{k^2}{2m^2} + h\right) - 1 \right] \tilde{\phi}(k) dk = 0, \quad (14)$$

which has a solution $\tilde{\phi}(k) \neq 0$ if equation

$$\zeta\left(\frac{-k^2}{2m^2} + h\right) = 1, \quad (15)$$

is satisfied. Taking usual relation $k^2 = -k_0^2 + \vec{k}^2 = -M^2$ equation (15) in the form

$$\zeta\left(\frac{M^2}{2m^2} + h\right) = 1, \quad (16)$$

determines mass spectrum $M^2 = \mu_h m^2$, where set of values of spectral function μ_h depends on h .

Equation (16) gives infinitely many tachyon mass solutions. Namely, function $\zeta(s)$ is continuous for real $s \neq 1$ and changes sign crossing its zeros $s = -2n$, $n \in \mathbb{N}$. According to relation $\zeta(1-2n) = -B_{2n}/(2n)$ and values of the Bernoulli numbers ($B_0 = 1$, $B_1 = -1/2$, $B_2 = 1/6$, $B_4 = -1/30$, $B_6 = 1/42$, $B_8 = -1/30$, $B_{10} = 5/66$, $B_{12} = -691/2730$, $B_{14} = 7/6$, $B_{16} = -3617/510$, $B_{18} = 43867/798$, \dots) it follows that $|\zeta(1-2n)| = |B_{2n}/(2n)| > 1$ if and only if $n \geq 9$. Taking into account also regions where $\zeta(1-2n) > 0$ we conclude that $\zeta(s) = 1$ has two solutions when $-20 - 4j < s < -18 - 4j$ for every $j = 0, 1, 2, \dots$. Consequently, for any $h \in \mathbb{Z}$, we obtain infinitely many tachyon masses M^2 :

$$M^2 = -(40 + 8j + 2h - a_j) m^2 \quad \text{and} \quad M^2 = -(36 + 8j + 2h + b_j) m^2, \quad (17)$$

where $a_j \ll 1$, $b_j \ll 1$ and $j = 0, 1, 2, \dots$.

3 Discussion with respect to some h

Among formally infinitely many possible values of h in (5) we are going now to consider five of them ($h = 0$, $h = \pm 1$ and $h = \pm 2$), which seem to be the most interesting. The case $h = -2$ is the simplest form of coefficients C_n which contain $n - 1$. In the case $h = -1$, coefficients $C_n = \frac{n-1}{n} \rightarrow 1$ for large n and Lagrangians \mathcal{L}_n are taken into account almost at an equal footing. In the third case, coefficients $C_n = \frac{n-1}{n^2}$ are inverses of those within \mathcal{L}_n , and considerably simplify obtained expressions. Cases $h = 1$ and $h = 2$ are taken into account to have a more complete insight about behavior of the Lagrangian L_h around $h = 0$.

3.1 Case $h = -2$

The Lagrangian (10), the corresponding potential and equation of motion now read respectively:

$$L_{-2} = \frac{m^D}{g^2} \left[-\frac{1}{2} \phi \zeta \left(\frac{\square}{2m^2} - 2 \right) \phi + \frac{2\phi^2 - \phi}{(1 - \phi)^2} - \frac{1}{2} \ln(1 - \phi)^2 \right], \quad (18)$$

$$V_{-2}(\phi) = \frac{m^D}{g^2} \left[\frac{\phi - 2\phi^2}{(1 - \phi)^2} + \frac{1}{2} \ln(1 - \phi)^2 \right], \quad (19)$$

$$\zeta \left(\frac{\square}{2m^2} - 2 \right) \phi = \frac{1}{(2\pi)^D} \int_{\mathbb{R}^D} e^{ixk} \zeta \left(-\frac{k^2}{2m^2} - 2 \right) \tilde{\phi}(k) dk = \frac{\phi(\phi + 1)}{(1 - \phi)^3}. \quad (20)$$

Potential $V_{-2}(\phi)$ has one local minimum $V_{-2}(-1) \approx -0.057 \frac{m^D}{g^2}$ and one local maximum $V_{-2}(0) = 0$. It is singular at $\phi = 1$ (i.e. $\lim_{\phi \rightarrow 1} V_{-2}(\phi) = -\infty$) and $\lim_{\phi \rightarrow \pm\infty} V_{-2}(\phi) = +\infty$. Equation of motion (20) has two trivial solutions: $\phi(x) = 0$ and $\phi(x) = -1$. Solution $\phi(x) = -1$ can be also shown taking $\tilde{\phi}(k) = -\delta(k) (2\pi)^D$ and $\zeta(-2) = 0$ in (20).

3.2 Case $h = -1$

Respectively, the corresponding Lagrangian, potential and equation of motion are:

$$L_{-1} = \frac{m^D}{g^2} \left[-\frac{1}{2} \phi \zeta \left(\frac{\square}{2m^2} - 1 \right) \phi + \frac{\phi}{1-\phi} + \frac{1}{2} \ln(1-\phi)^2 \right], \quad (21)$$

$$V_{-1}(\phi) = \frac{m^D}{g^2} \left[\frac{\zeta(-1)}{2} \phi^2 - \frac{\phi}{1-\phi} - \frac{1}{2} \ln(1-\phi)^2 \right], \quad (22)$$

$$\zeta \left(\frac{\square}{2m^2} - 1 \right) \phi = \frac{1}{(2\pi)^D} \int_{\mathbb{R}^D} e^{ixk} \zeta \left(-\frac{k^2}{2m^2} - 1 \right) \tilde{\phi}(k) dk = \frac{\phi}{(1-\phi)^2}, \quad (23)$$

where $\zeta(-1) = -\frac{1}{12}$.

This potential has the following properties: local maximum $V_{-1}(0) = 0$, $\lim_{\phi \rightarrow 1-0} V_{-1}(\phi) = -\infty$, $\lim_{\phi \rightarrow 1+0} V_{-1}(\phi) = +\infty$, $\lim_{\phi \rightarrow \pm\infty} V_{-1}(\phi) = -\infty$ and there is no stable vacuum. The equation of motion (23) has a constant trivial solution only for $\phi(x) = 0$.

3.3 Case $h = 0$

The related Lagrangian is

$$L_0 = -\frac{m^D}{g^2} \left[\frac{1}{2} \phi \zeta \left(\frac{\square}{2m^2} \right) \phi + \phi + \frac{1}{2} \ln(1-\phi)^2 \right]. \quad (24)$$

The corresponding potential is

$$V_0(\phi) = \frac{m^D}{g^2} \left[\frac{\zeta(0)}{2} \phi^2 + \phi + \frac{1}{2} \ln(1-\phi)^2 \right], \quad (25)$$

where $\zeta(0) = -\frac{1}{2}$. It has two local maxima: $V_0(0) = 0$ and $V_0(3) \approx 1.443 \frac{m^D}{g^2}$. There are no stable points and $\lim_{\phi \rightarrow 1} V_0(\phi) = -\infty$, $\lim_{\phi \rightarrow \pm\infty} V_0(\phi) = -\infty$.

The equation of motion is

$$\zeta \left(\frac{\square}{2m^2} \right) \phi = \frac{1}{(2\pi)^D} \int_{\mathbb{R}^D} e^{ixk} \zeta \left(-\frac{k^2}{2m^2} \right) \tilde{\phi}(k) dk = \frac{\phi}{1-\phi}. \quad (26)$$

It has two solutions: $\phi = 0$ and $\phi = 3$. The solution $\phi = 0$ is evident. The solution $\phi = 3$ follows from the Taylor expansion of the Riemann zeta function operator

$$\zeta\left(\frac{\square}{2m^2}\right) = \zeta(0) + \sum_{n \geq 1} \frac{\zeta^{(n)}(0)}{n!} \left(\frac{\square}{2m^2}\right)^n, \quad (27)$$

as well as from $\tilde{\phi}(k) = (2\pi)^D 3 \delta(k)$.

3.4 Case $h = 1$

Analogously to the previous cases, one has

$$L_1 = \frac{m^D}{g^2} \left[-\frac{1}{2} \phi \zeta\left(\frac{\square}{2m^2} + 1\right) \phi + \phi + \frac{1-\phi}{2} \ln(1-\phi)^2 \right], \quad (28)$$

$$V_1(\phi) = \frac{m^D}{g^2} \left[\frac{\zeta(1)}{2} \phi^2 - \phi - \frac{1-\phi}{2} \ln(1-\phi)^2 \right], \quad (29)$$

$$\frac{1}{(2\pi)^D} \int_{\mathbb{R}^D} e^{ixk} \zeta\left(-\frac{k^2}{2m^2} + 1\right) \tilde{\phi}(k) dk = -\frac{1}{2} \ln(1-\phi)^2, \quad (30)$$

where $\zeta(1) = \infty$ gives $V_1(\phi) = \infty$.

3.5 Case $h = 2$

$$L_2 = \frac{m^D}{g^2} \left[-\frac{1}{2} \phi \zeta\left(\frac{\square}{2m^2} + 2\right) \phi - \frac{1-\phi}{2} \ln(1-\phi)^2 - \phi - \phi \int_0^\phi \frac{\ln(1-w)^2}{2w} dw \right]. \quad (31)$$

$$V_2(\phi) = \frac{m^D}{g^2} \left[\frac{\zeta(2)}{2} \phi^2 + \frac{1-\phi}{2} \ln(1-\phi)^2 + \phi + \phi \int_0^\phi \frac{\ln(1-w)^2}{2w} dw \right]. \quad (32)$$

$$\frac{1}{(2\pi)^D} \int_{\mathbb{R}^D} e^{ixk} \zeta\left(-\frac{k^2}{2m^2} + 2\right) \tilde{\phi}(k) dk = - \int_0^\phi \frac{\ln(1-w)^2}{2w} dw. \quad (33)$$

Since holds equality $-\int_0^1 \frac{\ln(1-w)}{w} dw = \sum_{n=1}^{\infty} \frac{1}{n^2} = \zeta(2)$ one has trivial solution $\phi = 1$ in (33).

4 Concluding remarks

As a first step towards construction of an effective field theory for adelic open scalar string, we have derived a few Lagrangians which contain all corresponding n -adic Lagrangians ($n \in \mathbb{N}$). As a result one obtains that an infinite number of spacetime derivatives and related nonlocality are governed by the Riemann zeta function. Potentials are nonpolynomial. Tachyon mass spectra are determined by definite equations. p -Adic Lagrangians can be easily restored from a zeta Lagrangian using just an inverse procedure for its construction.

This paper contains some basic classical properties of the introduced zeta scalar field. There are still many classical aspects which should be investigated. One of them is a systematic study of the equations of motion and nontrivial solutions. In the quantum sector it is desirable to investigate scattering amplitudes and make comparison with adelic string.

Using the above procedure there is a sense to consider construction of a Lagrangian for an open-closed zeta string starting from p -adic ones presented in [2]. Note that effective Lagrangian for open-closed p -adic strings was also used to analyze tachyon condensation [27].

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